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Technical Notes

Note on Parameterization of Airfoils

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Nomenclature

S_i = Bernstein polynomial
x = coordinate measured along the chord line
Z_c = camber as a function of x
Z_l = lower surface z coordinate
Z_t = thickness as a function of x
Z_u = upper surface z coordinate
z = coordinate orthogonal to x

I. Introduction

HISTORICALLY, airfoils have been parameterized in a wide variety of ways. Two popular but closely related methods have been the class/shape function transformation (CST) method [1] (a generalization of the F function method developed by Kulfan in the 1970's) and a classical camber/thickness parameterization that has been used for wing design using optimization with TRANAIR for 15 years [2] developed over the years with input from Boeing design engineers. We will show that parameterizing the upper and lower surfaces using a square root singularity at the leading edge as in [1] must in general lead to a discontinuity in curvature at the leading edge if the shape function is bounded unless the first derivative of the camber is zero at the leading edge. Further, as can be seen from classical approximation theory, the ability to approximate airfoils with smooth camber at the leading edge will suffer unless all derivatives of the camber are zero at the leading edge. However, as discussed in [3], a more general method perhaps better matched to modern optimization methods is to parameterize airfoils using X and Z as functions of arc length. It appears that there are advantages to parameterizing the curvature of the airfoil as a function of the arc length for aerodynamic optimization.

II. Results

For this discussion, we consider only two dimensional airfoils with blunt leading edge and sharp trailing edges and we take the X coordinate as along the chord line of the airfoil and the Z coordinate as the other coordinate. The upper and lower surfaces will be designated by subscripts u and l. For simplicity, we will assume the airfoil to have unit chord length. As defined in [1], the CST method is given by

$$Z_u(x) = \sqrt{x}(1-x)\sum_{i=1}^{n} Au_i S_i(x)$$

$$Z_l(x) = \sqrt{x}(1-x)\sum_{i=1}^{n} Al_i S_i(x)$$

where the S_i are Bernstein polynomials of order $n-1$ and the Au_i are coefficients for the upper surface and Al_i are coefficients for the lower surface. The camber and thickness decomposition used by wing designers uses the camber and thickness defined by

$$Z_c(x) = \frac{1}{2}(Z_u(x) + Z_l(x)) \qquad Z_t(x) = Z_u(x) - Z_l(x)$$

In the CST method, the upper and lower surfaces are approximated. In the camber and thickness method, the camber and thickness are instead approximated directly:

$$Z_c(x) = \sum_{i=1}^{n} c_i B_i(x) \qquad Z_t(x) = \sqrt{x}\sum_{i=1}^{m} t_i B_i(x)$$

where the B_i are typically spline functions of order 5. However, orthonormal polynomials have also been used.

On the other hand, as can been seen from the definitions, the derived camber and thickness functions for CST are given by

$$\mathrm{CST}_c(x) = \frac{1}{2}\sqrt{x}(1-x)\sum_{i=1}^{n}(Al_i + Au_i) * S_i(x)$$

$$\mathrm{CST}_t(x) = \sqrt{x}(1-x)\sum_{i=1}^{n}(Au_i - Al_i) * S_i(x)$$

Thus, if the camber Z_c is a given smooth function, the approximation problem for CST is find Al_i and Au_i such that $Z_c = \frac{1}{2}\sqrt{x}(1-x)\sum_{i=1}^{n}(Al_i + Au_i) * S_i(x)$. For any smooth basis S_i, the error will be substantially degraded in the usual error norms, unless the value and the all the derivatives of Z_c are zero at the leading edge, in which case the $\sqrt{x}$ drops out of the CST camber equation.

By contrast, in the preceding camber and thickness parameterization, any smooth camber distribution can be approximated and the error will converge at the usual rate determined by the basis functions B_i.

This can be viewed in the following way. If the upper and lower surfaces are parameterized separately using a $\sqrt{x}$ singular function as in [1], then if the radius of curvature at the leading edge is the same for the upper and lower surfaces, the camber and its first derivative must be zero at the leading edge. This can be seen by considering simple shape functions. In the limit near the leading edge it suffices to consider a simple model problem. Let the camber be $\mathrm{CST}_c(x) = a\sqrt{x}$ and the thickness be $\mathrm{CST}_t(x) = \sqrt{x}$. Now, the upper and lower surfaces are given by $Z_u(x) = (a + \frac{1}{2})\sqrt{x}$ and $Z_l(x) = (a - \frac{1}{2})\sqrt{x}$ which is in the form required by CST. Computing the curvature for these two functions at $x = 0$ gives $\mathrm{CRV}_u = 2/c_u^2$ where $c_u = a + \frac{1}{2}$. For the lower surface, $\mathrm{CRV}_l = 2/c_l^2$ where $c_l = a - \frac{1}{2}$. It is easy to see that $\mathrm{CRV}_u = \mathrm{CRV}_l$ only if $a = 0$. A power series analysis for an arbitrary smooth camber function about $x = 0$ now shows that the first derivative of the camber must be zero at the leading edge.

Finally, one can extend easily the camber/thickness method parameterization to more general situations, for example when the upper and lower surface movements are restricted to different regions of the airfoil.

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